Numerical Simulation of Freak Waves: Detection, Predict and Breaking

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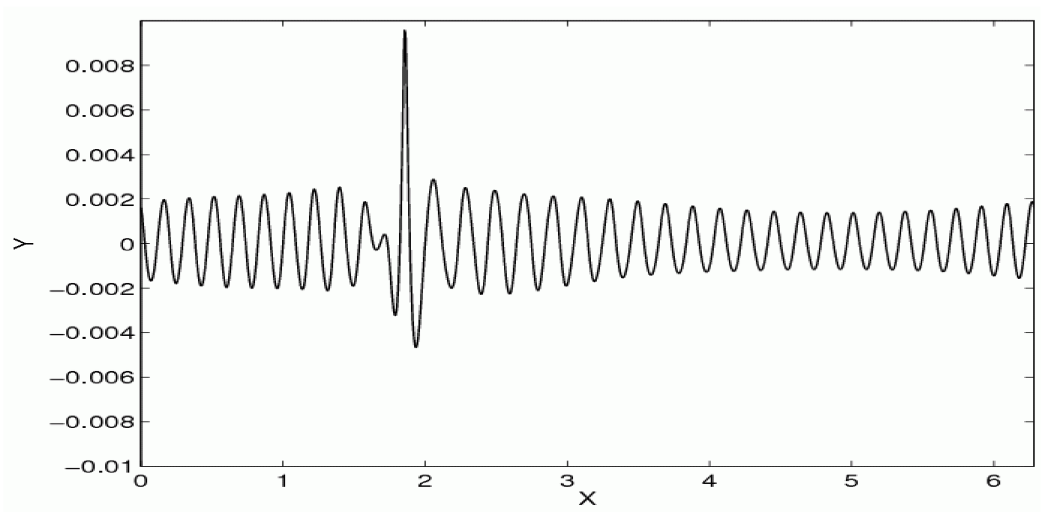
Introduction

We consider the exact numerical simulation of nonlinear water waves on the basis of nonlinear differential equations in conformal variables. Let us note that these equations are equivalent to the Euler equations for the area with free boundary. Also using of the differential inclusions allows considering the external influences and the numerical errors. In our numerical experiments the freak waves have been studied.



Main Goals:

- **Detection:** detection of freak waves during the numerical experiments
- **Predict:** predict of freak waves in the numerical experiments
- **Breaking:** breaking of freak waves by means of a blow to surface or other external influences

Basic Equation:

We use following equations the dynamic equations in conformal variables:

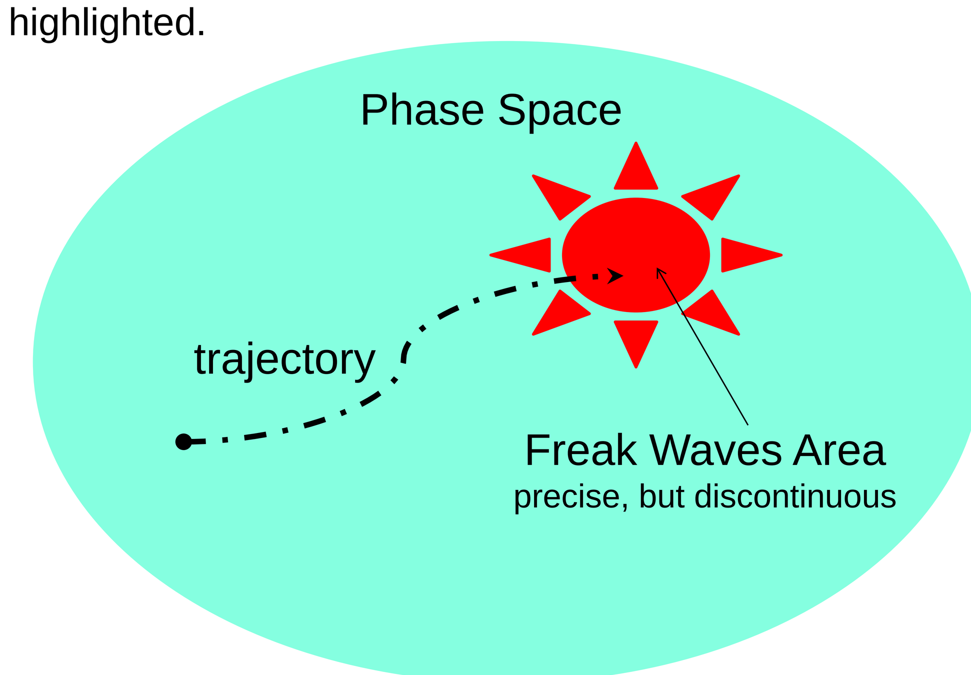
$$R_t(t,u) = i(UR_u - U_u R)$$
$$V_t(t,u) = i(UV_u - B_u R) + g(R-1)$$

where U=P(VR*+RV*), B=P(VV*), P=1/2(I+iH), H is the Hilbert operator. Unknown functions in this system are functions R (t, u) and V (t, u) which allows to restore a wave in accuracy.

These equations have been obtained in article [Dyachenko2001]. Strict mathematical results on the resolvability of this system and the description of methods of its numerical simulation are presented in works [Shamin2008, Shamin2010]. In work [Zakharov and al. 2010] these equations were applied to research of freak waves. The similar equations were used in works [Chalikov&Sheinin2005], [Ruban2005] and others.

Detection

We consider dynamic system which describes dynamics of surface waves. Thus in phase space the area of freak waves is highlighted.



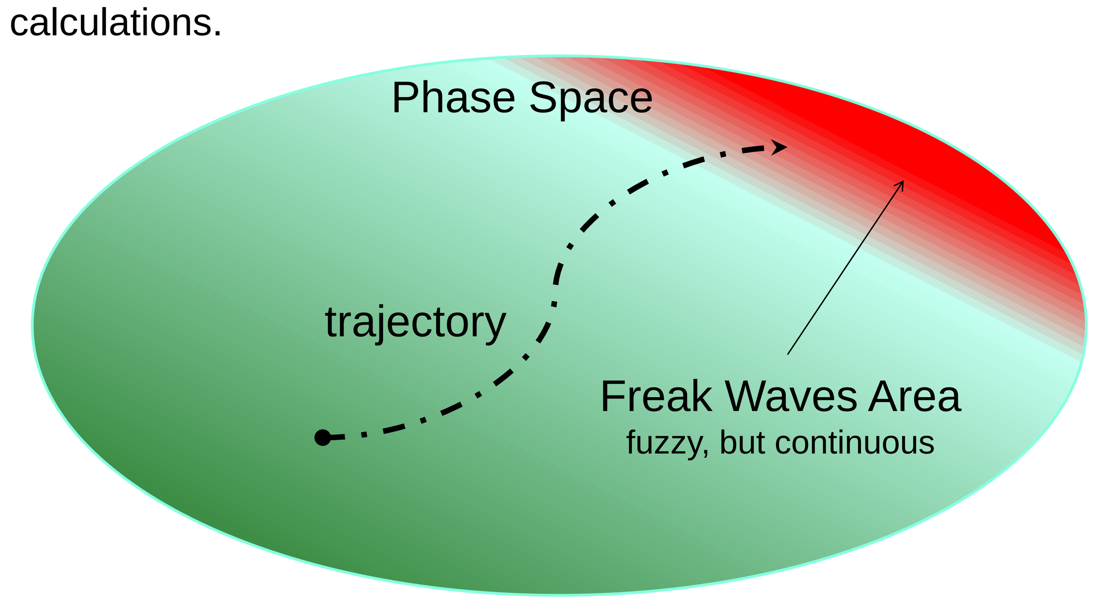
This area is defined on the basis of functional FW. This functional is defined on the phase space. However the standard amplitude criterion is defined by discontinuous function al.

In computing experiments it is very important to have continuous functional. In our experiments we have proposed other criterion of freak waves.

The new criterion is based on functional which is calculated on complete trajectory. After the termination of each experiment, the value ν was calculated by the formula

$$\nu = \frac{\max y(x,t)}{\langle |y| \rangle}, \quad \langle |y| \rangle = \frac{1}{T}\int_0^T \max_{x\in[0,2\pi]} |y(x,t)|\, dt$$

Here, the maximum in numerator is taken on coordinate and on time in the interval $0 < t < T$. The freak wave was fixed if the parameter ν exceeded the critical value ν=1.8. It was required also that the local steepness of the wave exceeded critical value (≤0.3). This requirement is caused by obvious physical reasons and is rather essential. This definition quantitatively does not differ essentially from the standard definition where it is considered that freak waves twice exceed significant wave height, but new criterion is steady concerning errors of calculations.



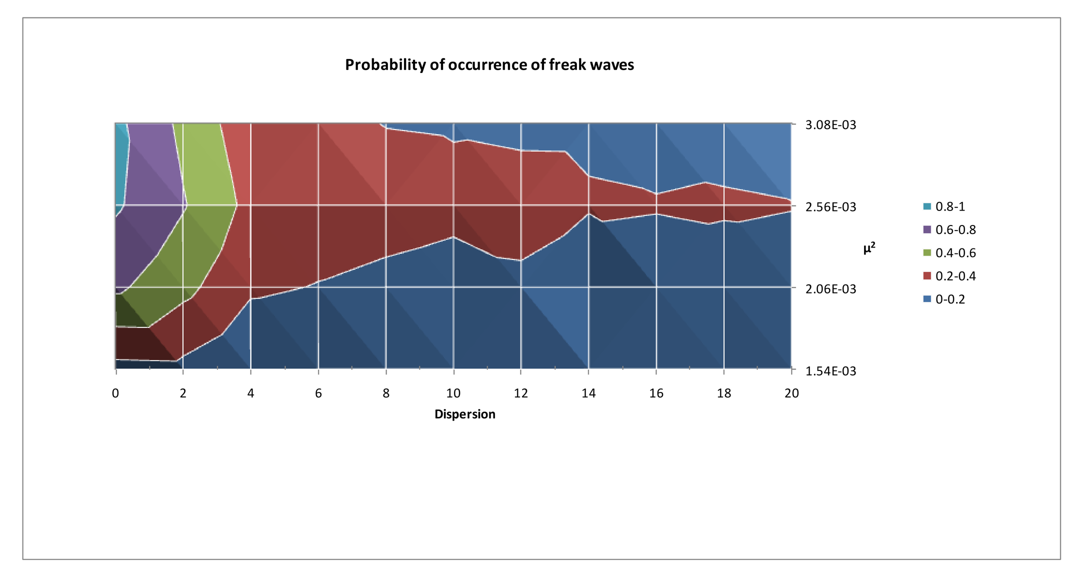
Predict

The prediction of occurrence of freak waves is the very important problem. For the decision of this problem we calculate probability of occurrence of the freak waves.

In our experiments, the initial conditions were defined as an ensemble of waves traveling in the same direction with the average wavenumber K_0 = 25. We assumed that initial perturbation of the surface is set by the sum of harmonics with random phases

$$y_0(x) = \sum_{-1/2K\max}^{1/2K\max} \phi(k-K_0)\cos(kx-\xi_k)$$

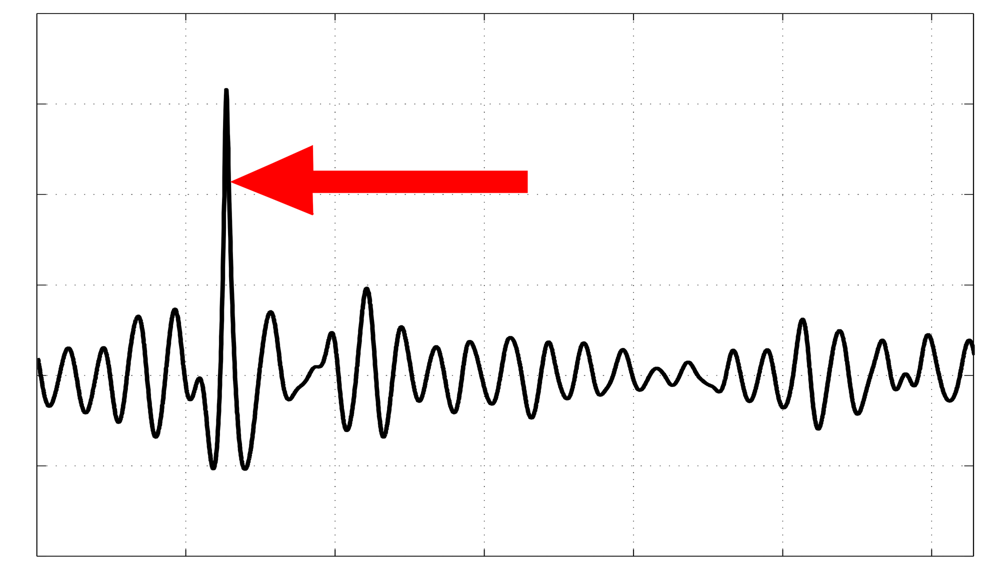
Here, K_{max} is the total number of spectral modes and ξ_k is the random variable. The initial velocities were assumed to relate by formulas of the linear theory. Conformal mapping was carried out by means of the iterative algorithm offered by A.I. Dyachenko. The function φ(k) was defined by formula $\varphi(k)=\kappa \exp(-\alpha k^2)+\delta_k$, $|k|\le K_w$, $\varphi(k)= \delta_k$, $|k|>K_w$. Here, δ_k are the independent random parameters. The number $1\le K_w \le 0$ defines the spectral width, κ, α are "internal" parameters of the spectrum. They are defined so that external parameters accept the preset values: average steepness μ and dispersion D. Further, we calculate the exact values of the total energy E and we observe that the contribution of random noise is no more than three percent. 5000 individual experiments have been done. In each experiment time varied in the range of 0<t<200 that corresponded approximately to 500 periods of waves. If there was a collapse of waves, the experiment was stopped ahead of time. In calculations the full number of harmonics was K_{max}=2048 or K_{max}=4096 depending on the total energy, which varied within $1.5\ 10^{-4} \le E \le 4\ 10^{-4}$.



In this picture the pattern of probability of occurrence of freak waves is presented. It follows from our data that even for waves of enough moderate steepness ($\mu^2=2.06\ 10^{-3}$, μ=0.045), the formation of an extreme wave for a time interval as short as 500 periods (at the period of 10 seconds it is less than one and a half hours) is a rather probable event even if the spectral width on wavenumbers is comparable with the carrier wavenumber. Actually, this experiment underlines the fact that the formation of extreme waves is an ordinary event.

Breaking

The problem about destruction of freak waves is very interesting and topical. We assume that the freak wave can be destroyed, having struck on its surface. It is schematically presented in a following drawing.



Such approach is connected with questions on stability of freak waves. Results of recent numerical experiments have shown stability of these waves! Therefore the problem of destruction of freak waves is difficult. Our model allows to consider a blow on a surface of wave. Now we carry out computing experiments for the purpose of destruction of waves at the moment of their formation.

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For further information

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